jets give momenta to the ejected exhaust gases. They in turn gain momentum in the forward direction. For a rocket with no external forces on it, the increase in momentum during each second will depend on how much gas is ejected, which equals the mass of fuel burned, and on the speed of the gas (Figure 3.9).

Why is linear momentum conserved? Because of Newton's second and third laws of motion. When two objects exert forces on each other by colliding or via a spring plunger, the forces are equal and opposite. They push on each other with the same size force but in opposite directions. By the second law (alternate form), these equal forces cause the momenta of both objects to change at the same rate. As long as the objects are interacting, they change one another’s momentum by the same amount but in opposite directions. The momentum gained (or lost) by one object is exactly offset by the momentum lost (or gained) by the other. The total linear momentum is not changed. In Example 3.2 the 1,000-kilogram car is slowed from 10 m/s before the collision to 4 m/s after the collision (Figure 3.5). Its momentum is decreased by 6,000 kg·m/s (1,000 kilograms × 10 m/s − 1,000 kilograms × 4 m/s). The 1,500-kilogram car goes from 0 m/s before to 4 m/s after. So its momentum is increased by 6,000 kg·m/s (1,500 kilograms × 4 m/s).

These examples illustrate the usefulness of conservation laws. The approach is different from that used with Newton's second law in Chapter 2, where it was necessary to know the size of the force that acts on the object at each moment to determine its velocity. With the law of conservation of linear momentum, we do not have to know the details of the interactions—how large the forces are and how long they act. All we need is some information about the system before and after the interaction. Note that in Example 3.2 we used information from after the collision to determine the speed of the car before the collision. In the example with the carts, we determined the ratio of the speeds after the interaction.

### DO-IT-TOGETHER PHYSICS

The next time you go skateboarding or skating (ice, roller, or in-line) with a friend, experiment with the conservation of linear momentum in various kinds of collisions. Try to reproduce the collisions illustrated in Figures 3.5, 3.7, and 3.12. You can also experiment with Newton's third law of motion. See Figure 2.33 for examples.

### 3.3 WORK: THE KEY TO ENERGY

The law of conservation of energy is arguably the most important of the conservation laws. Not only is it useful for solving problems, it is a powerful theoretical statement that can be used to understand widely diverse phenomena and to show what hypothetical processes are or are not possible. As we mentioned earlier, the concept of energy is one of the most important in physics. This is because energy takes many forms and is involved in all physical processes. One could say that every interaction in our universe involves a transfer of energy or a transformation of energy from one form to another.

One might compare the concept of energy to that of financial assets, which can take the form of cash, real estate, material goods, or investments, among other things. The study of economics is in part a study of these forms of financial assets and how they are transferred and transformed. Much of physics deals with the forms of energy and the transformations that occur during interactions.

When first encountered, the concept of energy is a bit difficult to understand because there is no simple way to define it. As an aid, we will first introduce work, a physical quantity that is quite basic and which gives us a nice foundation for understanding energy.

The idea of work in physics arises naturally when one considers simple machines like the lever and the inclined plane when used in situations with negligible friction. Let's say that you use a lever to raise a heavy rock (Figure 3.10). If you place the fulcrum close to the rock, you find that a small, downward force on your end results in a larger, upward force on the rock. However, the distance your end moves as you push it down is correspondingly larger than the distance the rock is raised. By measuring the forces and
When a lever is used to raise a rock, a small force on the right end results in a larger force on the left end. But the right end moves a greater distance than the left end. The force multiplied by the distance moved is the same for both ends. Distances, we find that dividing the larger force by the smaller force gives the same number (or ratio) as dividing the larger distance by the smaller distance. In particular:

\[
\frac{F \text{ on left end}}{F \text{ on right end}} = \frac{d \text{ right end moves}}{d \text{ left end moves}}
\]

We can multiply both sides by \( F \text{ on right end} \) and \( d \text{ left end moves} \) and get the following result:

\[
(F \text{ on left}) \times (d \text{ left moves}) = (F \text{ on right}) \times (d \text{ right moves})
\]

\[
F_{\text{left}} \cdot d_{\text{left}} = F_{\text{right}} \cdot d_{\text{right}}
\]

In other words, even though the two forces and the two distances are different, the quantity force times distance has the same value for both ends. We might say that raising the rock is a fixed task. One can perform the task by lifting the rock directly or by using a lever. In the former case, the force is large, equal to the rock’s weight, but the distance moved is small. When using the lever, the force is smaller, but the distance is larger. The quantity \( Fd \) is the same, regardless of which way the task is accomplished.

We reach the same conclusion when considering an inclined plane. Let us say that a barrel must be placed on a loading dock (Figure 3.11). Lifting the barrel directly requires a large force acting through a small distance, the height of the dock. If the barrel is rolled up a ramp, a smaller force is needed, but the barrel moves a greater distance. Again, the product of the force and the distance moved is the same for the two methods.

\[
(F \text{ lifting}) \times \text{ height} = (F \text{ rolling}) \times \text{ (ramp length)}
\]

\[
F_{\text{lifting}} \cdot d_{\text{lifting}} = F_{\text{rolling}} \cdot d_{\text{rolling}}
\]

The quantity force times distance is obviously a useful way of measuring the “size” of a task. It is called work.
**Work**  The force that acts times the distance moved in the direction of the force:

\[ \text{work} = Fd \]

<table>
<thead>
<tr>
<th>Physical Quantity</th>
<th>Metric</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work</td>
<td>joule (J) [SI unit]</td>
<td>foot-pound (ft-lb)</td>
</tr>
<tr>
<td></td>
<td>erg</td>
<td>British thermal unit (Btu)</td>
</tr>
<tr>
<td></td>
<td>calorie (cal)</td>
<td>kilowatt-hour (kW h)</td>
</tr>
</tbody>
</table>

Since force is a vector, *work equals the distance moved times the component of the force parallel to the motion*. Work itself is not a vector. There is no direction associated with work.

Whenever an object moves and there is a force acting on the object in the same or opposite direction that it moves, work is done. When the force and the motion are in the same direction, the work is positive. When they are in opposite directions, the work is negative.

**EXAMPLE 3.3**  Because of friction, a constant force of 100 newtons is needed to slide a box across a room (Figure 3.12). If the box moves 3 meters, how much work is done?

\[ \text{work} = Fd \]
\[ = 100 \text{ N} \times 3 \text{ m} \]
\[ = 300 \text{ N-m} \]

The unit in the answer is the newton-meter (N-m). This is called the *joule*.

1 joule = 1 newton-meter = 1 newton \times 1 meter

1 J = 1 N-m

The joule is a derived unit of measure, and it is the SI unit of work and energy. Just remember that if you use only SI units for force, distance, and so on, your unit for work and energy will always be the joule (J).

**EXAMPLE 3.4**  Let's say that the barrel in Figure 3.11 has a mass of 30 kilograms and that the height of the dock is 1.2 meters. How much work would you do when lifting the barrel?

\[ \text{work} = Fd \]
The force is just the weight of the barrel, \(mg\).
\[
F = W = mg = 30 \text{ kg} \times 9.8 \text{ m/s}^2 = 294 \text{ N}
\]
Hence:
\[
\text{work} = Fd = Wd = 294 \text{ N} \times 1.2 \text{ m} = 353 \text{ J}
\]
The work you do when rolling the barrel up the ramp would be the same. The force would be smaller, but the distance would be larger.

It is important to note that work is not done if the force is perpendicular to the motion. When you simply carry a box across a room, your force on the box is vertical, while the motion of the box is horizontal. Hence, you do no work on the box (Figure 3.13).

Uniform circular motion is another situation in which a force acts on a moving body but no work is done. Recall that a centripetal force must act on anything to keep it moving along a circular path (Figure 3.14). This force is always toward the center of the circle and perpendicular to the object’s velocity at each instant. Therefore, the force does not do work on the object.

Work can be done in a circular motion by a force that is not a centripetal force. When you turn the crank on a pencil sharpener, for instance, you exert a force on the handle that is in the same direction as the handle’s motion. Hence, you do work on the handle.

Work is done on an object when it is accelerated in a straight line. The following example shows how the amount of work can be computed. (In the next section we will see that there is an easier way to do this.)

**EXAMPLE 3.5** In Example 2.2 we used Newton’s second law to compute the force needed to accelerate a 1,000-kilogram car from 0 to 27 m/s in 10 seconds. Our answer was \(F = 2,700\) newtons. How much work is done?

\[
\text{work} = Fd
\]
To find the distance that the car travels, we use the fact that the car accelerates at 2.7 m/s\(^2\) for 10 seconds. Using the equation from Section 1.5:
\[
d = \frac{1}{2}at^2 = \frac{1}{2} \times 2.7 \text{ m/s}^2 \times (10 \text{ s})^2 = 1.35 \text{ m/s}^2 \times 100 \text{ s}^2 = 135 \text{ m}
\]
So the work done is
\[
\text{work} = Fd = 2,700 \text{ N} \times 135 \text{ m} = 364,500 \text{ J}
\]
We have seen that work is done (a) when a force acts to move something against the force of gravity (Figures 3.10 and 3.11), (b) when a force acts to move something against friction (Figure 3.12), and (c) when a force accelerates an object. There are many other possibilities. When a force distorts something, work is done. For example, to compress or stretch a spring, a force must act on it. This force acts through a distance in the same direction as the force, so work is done.

Work is also done when a force causes something to slow down. When you catch a ball, your hand exerts a force on the ball. As the ball slows down, it pushes your hand back with an equal and opposite force (see Figure 3.15). In this case, the ball does work on your hand. Your hand does negative work on the ball. The amount of work that the ball does on your hand is equal to the amount of work that was originally done on the ball to accelerate it (ignoring the
As you catch a ball, your hand exerts a force on the ball. By Newton's third law, the ball exerts an equal and opposite force on your hand. This force does work on you as you slow down the ball. The work done is equal to the work that was done to accelerate the ball in the first place.

Effect of air resistance. If you allow your hand to move back as you catch the ball, the force of the ball on you will be less than if you try to keep your hand stationary. The work that the ball will do on your hand is the same either way. Since work = force × distance, the force on your hand will be smaller if the distance that the ball moves while you catch it is larger.

One last example: When an object falls freely, the force of gravity does work on it. As in the previous example with the car, this work goes to accelerate the object. In particular, if a body falls a distance \( d \), the work done on it by the force of gravity is

\[
\text{work} =Fd 
\]

But

\[
F = W = mg
\]

So:

\[
\text{work} = mgd
\]

The work that the force of gravity does on an object as it falls is equal to the work that was done to lift the object the same distance (Figure 3.16). When something is lifted, we say that work is done against the force of gravity. The movement is in the opposite direction of the force. When something falls, work is done by the force of gravity. The movement is in the same direction as the force.

In summary, work is done by a force whenever the point of application moves in the direction of the force. Work is done against the force whenever the point of application moves opposite the direction of the force. Forces always come in pairs that are equal and opposite (Newton's third law of motion). Consequently, when work is done by one force, this work is being done against the other force in the pair.

The work done lifting an object equals the work done by gravity as the object falls.
3.4 ENERGY

In Section 3.3, we saw that work can often be “recovered.” The work that is done when a ball is thrown is equal to the work done by the ball when it is caught. The work done when lifting an object against the force of gravity is equal to the work done by the force of gravity when the object falls. When work is done on something, it gains some energy. This energy can then be used to do work. A thrown ball is given energy. This energy is given up to do work when the ball is caught.

**Energy**  The measure of a system's capacity to do work. That which is transferred when work is done. Abbreviated \( E \).

The units of energy are the same as the units of work. Like work, energy is a scalar.

The more work that is done on something, the more energy it gains, and the more work it can do in return. We might say that energy is “stored work.” To be able to do work, an object must have energy. When you throw a ball, you are transferring some energy from you to the ball. The ball can then do work. In Figures 3.13 and 3.14, no energy is transferred because no work is done.

There are many forms of energy corresponding to the many ways in which work can be done. Some of the more common forms of energy are chemical, electrical, nuclear, and gravitational, as well as the energy associated with heat, sound, light, and other forms of radiation. Anything possessing any of these forms of energy is capable of doing work (see Figure 3.17).

In mechanics, there are two main forms of energy, which we can classify under the single heading of mechanical energy. Anything that has energy because of its motion or because of its position or configuration has mechanical energy. We refer to the former as kinetic energy and to the latter as potential energy.

**Kinetic Energy**  Energy due to motion. Energy that an object has because it is moving. Abbreviated \( KE \).

Anything that is moving has kinetic energy. The simplest example is an object moving in a straight line. The amount of kinetic energy an object has depends on its mass and speed. In particular:

\[
KE = \frac{1}{2}mv^2 \quad \text{(kinetic energy)}
\]
The kinetic energy that an object has is equal to the work done when accelerating the object from rest. So another way to determine the amount of work done when accelerating an object is to compute its kinetic energy. This also shows that the work done depends only on the object’s final speed and not on how rapidly or slowly it was accelerated.

**Example 3.6** In Example 3.5 we computed the work that is done on a 1,000-kilogram car as it accelerates from 0 to 27 m/s. The car’s kinetic energy when it is traveling 27 m/s is

\[
KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 1,000 \text{ kg} \times (27 \text{ m/s})^2 \\
= 500 \text{ kg} \times 729 \text{ m}^2/\text{s}^2 \\
= 364,500 \text{ J}
\]

This is the same as the work done as the car accelerates. The car can do 364,500 joules of work because of its motion.

The kinetic energy of a moving body is proportional to the square of its speed. If one car is going twice as fast as a second, identical car, the faster one has four times the kinetic energy. It takes four times as much work to stop the faster car. Note that the kinetic energy of an object can never be negative. (Why? Because \( m \) is always positive and \( v^2 \) is positive even when \( v \) is negative.)

Since speed is relative, kinetic energy is also relative. A runner on a moving ship has \( KE \) relative to the ship and a different \( KE \) relative to something at rest in the water.

Another way that an object can have kinetic energy is by rotating. To make something spin, work must be done on it. A dancer or skater performing a pirouette has kinetic energy (see **Figure 3.18**). A spinning top has kinetic energy, as do the

---

*For the mathematically inclined—the kinetic energy that a body has equals the work done to accelerate it from rest. In the case of constant acceleration, we can compute the work:

\[
\text{work} = Fd
\]

Now we use what we learned in Chapters 1 and 2. The force needed is \( F = ma \), and the distance traveled is \( d = \frac{1}{2}at^2 \). So:

\[
\text{work} = Fd = ma \times \frac{1}{2}at^2 = \frac{1}{2}m \times a^2 \times t^2 = \frac{1}{2}m \times (at)^2
\]

But \( \Delta t \) equals the speed \( \Delta \Delta t \) that the body has. So:

\[
\text{work} = \frac{1}{2}mv^2 = KE
\]
Earth, Moon, Sun, and other astronomical objects as they spin about their axes. The amount of kinetic energy that a spinning object has depends on its mass, its rotation rate, and the way its mass is distributed. Some devices and toys use rotational kinetic energy as a way to "store" energy. Figure 3.19 shows a toy car and a shaver that operate this way.

**Potential Energy** Energy due to an object’s position or orientation. Energy that a system has because of its configuration. Abbreviated PE.

The amount of potential energy that a system acquires is equal to the work done to put it in that configuration. When an object is lifted, it is given potential energy. It can use this energy to do work. For example, when the weights on a cuckoo clock or any other gravity-powered clock are raised, they are given potential energy (Figure 3.20). As they slowly fall, they do work in operating the clock.

In Section 3.3 we computed the work done when an object is lifted. This is equal to the potential energy that it is given. Since the work is done against the force of gravity, it is called gravitational potential energy:

\[
PE = \text{work done} = \text{weight} \times \text{height} = Wd = mgd
\]

\[
PE = mgh \quad \text{(gravitational potential energy)}
\]

Gravitational potential energy is the most common type of potential energy, and it is often referred to simply as potential energy. Potential energy is a relative quantity because height can be measured relative to different levels.

**Example 3.7** A 3-kilogram brick is lifted to a height of 0.5 meters above a table (Figure 3.21). Its potential energy relative to the table is

\[
PE = mgh
\]

\[
PE = 3 \, \text{kg} \times 9.8 \, \text{m/s}^2 \times 0.5 \, \text{m}
\]

\[
= 14.7 \, \text{J} \quad \text{(relative to the table)}
\]

But the tabletop itself may be 1 meter above the floor. The brick's height above the floor is 1.5 meters, and so its potential energy relative to the floor is

\[
PE = mgh
\]

\[
PE = 3 \, \text{kg} \times 9.8 \, \text{m/s}^2 \times 1.5 \, \text{m}
\]

\[
= 44.1 \, \text{J} \quad \text{(relative to the floor)}
\]
A person sitting in a chair has potential energy relative to the floor, to the basement of the building, and to the level of the oceans. Usually, some convenient reference level is chosen for determining potential energies. In a room it is logical to use the floor as the reference level for measuring heights and, consequently, potential energies.

An object's potential energy is negative when it is below the chosen reference level. Often the reference level is chosen so that negative potential energies signify that an object cannot "leave." For example, on flat ground outdoors it is logical to measure potential energy relative to the ground level. Anything on the ground has zero potential energy, and anything above the ground has positive potential energy. If there is a hole in the ground, any object in the hole will have negative potential energy (Figure 3.22). Any object that has zero or positive potential energy can move about horizontally if it has any kinetic energy. Objects with negative potential energy are confined to the hole. They must be given enough energy to get out of the hole before they can move horizontally.

Springs and rubber bands can possess another type of potential energy: elastic potential energy. Work must be done on a spring to stretch or compress it. This gives the spring potential energy (Figure 3.23). This "stored energy" can then be used to do work. The actual amount of potential energy a spring has depends on two things: how much it was stretched or compressed and how strong it is. In Figure 3.7 the combined kinetic energies of the carts after the spring is released equal the original elastic potential energy of the spring. A stronger spring would possess more potential energy and would give the carts more KE, so they would go faster.

A number of devices use elastic potential energy. Toy dart guns have a spring inside that is compressed by the shooter. When the trigger is pulled, the spring is released and does work on the dart to accelerate it. The potential energy of the spring is converted to kinetic energy of the dart. The bow and arrow operate this same way, with the bow acting as a spring. Windup devices such as wrist watches, toys, and music boxes use energy stored in springs to operate the mechanism. Usually the spring is in a spiral shape, but the principle is the same. Rubber bands provide lightweight energy storage in some toy airplanes and birds (Figure 3.24).

Another form of energy that is important in mechanical systems is internal energy. (Internal energy, heat, and temperature are discussed formally in Chapter 5. Basically, the internal energy of a substance is just the total energy of all the atoms and molecules in the substance. To raise something's temperature, to melt a solid, or to boil a liquid all require increasing the internal energy of the substance. Internal energy decreases when a substance's temperature decreases, when a liquid freezes, or when a gas condenses.) Internal energy is involved whenever there is kinetic friction. In Figure 3.12 the work that is done on the box is converted into internal energy because of the friction between the box and the floor. The result is that the temperatures of the floor and the box are raised (although not by much). As a car or a bicycle brakes to a stop, its kinetic energy is converted into internal energy in the brakes. Automobile disc brakes can become red hot under extreme braking. Meteors (shooting stars) are a spectacular example of kinetic energy being
converted into internal energy. As they enter the atmosphere at very high speed, the air resistance heats them enough to glow and melt. Gravitational potential energy can also be converted into internal energy. A box slowly sliding down a ramp has its gravitational potential energy converted into internal energy by friction. If you climb a rope and then slide down it, you can burn your hands severely as some of your potential energy is converted into internal energy because of the friction between your hands and the rope.

Internal energy can also be produced by internal friction when something is distorted. The work you do when stretching a rubber band, pulling taffy, or crushing an aluminum can generates internal energy. When you drop something and it doesn’t bounce, like a book, most of the energy the object had is converted into internal energy on impact.

Internal energy arising from friction, unlike kinetic energy and potential energy, usually cannot be recovered. Work done to lift a box and give it potential energy can be recovered as work or some other form of energy. Work done to slide a box across a floor becomes internal energy that is “lost” (made unavailable). In every mechanical process, some energy is converted into internal energy. It has been estimated that in the United States the annual financial losses associated with overcoming friction exceed $300 billion.

In summary, work always results in a transfer of energy from one thing to another, in a transformation of energy from one form to another, or both. In our earlier analogy in which we compared energy to financial assets, work plays the role of a transaction such as buying, selling, earning, or trading. These transactions can be used to increase or decrease the net worth of an individual or to convert one form of asset into another. Work done on a system increases its energy. Work done by the system decreases the energy of the system. Work done within the system results in one form of energy being changed into another.

**DO-IT-YOURSELF PHYSICS**

Take a paper clip and straighten one of the ends. Bend the clip back and forth several times, then touch the part that was bending to your cheek. You should be able to feel that the clip is warm. The work you do in bending the clip is converted into internal energy. You can get the same result by stretching a rubber band.

**3.5 THE CONSERVATION OF ENERGY**

In the preceding section we described several situations in which one form of energy was converted into another. These included a dart gun (potential energy in a spring converted to kinetic energy of the dart), a car braking (kinetic energy converted into internal energy), and a box sliding down an inclined plane (gravitational potential energy becoming internal energy). There are countless situations involving many other forms of energy.

Many devices in common use are simply energy converters. Some examples are listed in Table 3.1. Some of these actually involve more than one conversion. In a hydroelec-

<table>
<thead>
<tr>
<th>Device</th>
<th>Energy Conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light bulb</td>
<td>Electrical energy to radiant energy</td>
</tr>
<tr>
<td>Car engine</td>
<td>Chemical energy (in fuel) to kinetic energy</td>
</tr>
<tr>
<td>Battery</td>
<td>Chemical energy to electrical energy</td>
</tr>
<tr>
<td>Elevator</td>
<td>Electrical energy to gravitational potential energy</td>
</tr>
<tr>
<td>Generator</td>
<td>Kinetic energy to electrical energy</td>
</tr>
<tr>
<td>Electric motor</td>
<td>Electrical energy to kinetic energy</td>
</tr>
<tr>
<td>Solar cell</td>
<td>Radiant energy (light) to electrical energy</td>
</tr>
<tr>
<td>Flute</td>
<td>Kinetic energy (of air) to acoustic energy (of sound wave)</td>
</tr>
<tr>
<td>Nuclear power plant</td>
<td>Nuclear energy to electrical energy</td>
</tr>
<tr>
<td>Hydroelectric dam</td>
<td>Gravitational potential energy (of water) to electrical energy</td>
</tr>
</tbody>
</table>
tric dam (see Figure 3.25), the potential energy of the water behind the dam is converted into kinetic energy of the water. The moving water then hits a turbine (propeller), which gives it kinetic energy of rotation. The rotating turbine turns a generator, which converts the kinetic energy into electrical energy. Internal energy is an intermediate form of energy in both the car engine and the nuclear power plant. In a car engine, the chemical energy in fuel is first converted into internal energy as the fuel burns (explodes). The heated gases expand and push against pistons (or rotors in rotary engines). These in turn make a crankshaft rotate. In a nuclear power plant, nuclear energy is converted into internal energy in the reactor core. This internal energy is used to boil water into steam. The steam is used to turn a turbine, which turns a generator that produces electrical energy.

Internal energy is also a “wasted” by-product in all of the devices in Table 3.1. Any device that has moving parts has some kinetic friction. Some of the input energy is converted into internal energy by this friction. The generator, electric motor, car engine, and the electrical power plants all have unavoidable friction (Figure 3.26). In some of the devices, internal energy is produced because of the basic nature of the process. Over 95% of the electrical energy used by an incandescent light bulb is converted to internal energy, not usable light. Over 60% of the available energy in coal-fired and nuclear power plants goes to unused internal energy. We will investigate this further in later chapters.

Even though there are many different forms of energy and countless devices that involve energy conversions, the following law always holds.

**Law of Conservation of Energy**. Energy cannot be created or destroyed, only converted from one form to another. The total energy in an isolated system is constant.

To be an isolated system, energy cannot leave or enter the system. For a mechanical system, work cannot be done on the system by an outside force, nor can the system do work on anything outside of it.

This law means that energy is a commodity that cannot be produced from nothing or disappear into nothing. If work is being done or a form of energy is “appearing,” then energy is being used or converted somewhere. Unlike money, which you can counterfeit or burn, you cannot manufacture or eliminate energy.

The law of conservation of energy is both a practical tool and a theoretical tool. It can be used to solve problems, notably in mechanics, and it is a necessary condition that proposed models must satisfy. As an example of the latter, a theoretical astrophysicist may develop a model that explains how stars convert nuclear energy into heat and radiation. A first test of the validity of the model is whether or not energy is conserved.

We now illustrate the practical usefulness of the law by considering some mechanical systems. In each case, we assume that friction is negligible, so we do not have to take into account any conversion of potential energy or kinetic energy into internal energy.

The basic approach in using the law of conservation of energy is the same as that used with the corresponding law for linear momentum. If there is a conversion of energy in the
system from one form to another, the total energy before the conversion equals the total energy after the conversion.

\[
\text{total energy before} = \text{total energy after}
\]

A nice example of this is something we have encountered before (in Sections 1.5 and 2.6), the motion of a freely falling body. If an object is raised to a height \( d \), it has gravitational potential energy. If it is released, the object will fall and convert its potential energy into kinetic energy. It is a continuous process. As it falls, its potential energy decreases because its height decreases, while its kinetic energy increases because its speed increases. If it is falling freely, there is no air resistance, and the only two forms of energy are kinetic and gravitational potential. Energy conservation then means that the sum of the kinetic energy and the potential energy of the object is always the same (see \( \bullet \) Figure 3.27).

\[
E = KE + PE = \text{constant}
\]

We can use this fact to show how the object’s speed just before it hits the floor depends on the height \( d \). We do this by looking at the object’s energy just as it is dropped and then just before it hits. At the instant it is released, its kinetic energy is zero, because its speed is zero, and its potential energy is \( mgd \).

\[
E = KE + PE = 0 + PE
\]

\[
= PE = mgd \quad \text{(just released)}
\]

When it reaches the floor, at the instant before impact, its potential energy is zero. So:

\[
E = KE + PE = KE + 0
\]

\[
= KE = \frac{1}{2}mv^2 \quad \text{(just before impact)}
\]

These two quantities are equal since the amount of the object’s energy has not changed, only the form. The potential energy the object had when it was released equals the kinetic energy it has just before impact.

\[
\frac{1}{2}mv^2 = mgd
\]

Dividing both sides by \( m \) and multiplying by 2, we get:

\[
\nu^2 = 2gd
\]

\[
\nu = \sqrt{2gd} \quad \text{(speed after falling distance } d)\]

*This is the answer to Challenge 7 in Chapter 1.
**TABLE 3.2 Speed Versus Distance for a Freely Falling Body**

<table>
<thead>
<tr>
<th>A. SI Units</th>
<th>B. English Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (d) (m)</td>
<td>Speed (v) (m/s)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>4.4</td>
</tr>
<tr>
<td>2</td>
<td>6.3</td>
</tr>
<tr>
<td>3</td>
<td>7.7</td>
</tr>
<tr>
<td>4</td>
<td>8.9</td>
</tr>
<tr>
<td>5</td>
<td>9.9</td>
</tr>
<tr>
<td>10</td>
<td>14</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>100</td>
<td>44</td>
</tr>
</tbody>
</table>

*Note: Parts A and B are independent. The distances and speeds in A and B are not equivalent.*

**EXAMPLE 3.8** In September 1989, two people rode a "barrel" over Niagara Falls and survived. (Some of their predecessors over the years had not.) The height of the falls is about 53 meters. Estimate their speed when they hit the water at the bottom of the falls.

Assuming that the air resistance was too small to affect their motion appreciably, we use the preceding results.

\[
\begin{align*}
v &= \sqrt{2gd} = \sqrt{2 \times 9.8 \text{ m/s}^2 \times 53 \text{ m}} \\
&= \sqrt{1039 \text{ m}^2/\text{s}^2} \\
&= 32.2 \text{ m/s} \quad \text{(about 72 mph)}
\end{align*}
\]

The speed of a freely falling body does not depend on its mass. It only depends on how far it has fallen \((d)\) and the acceleration of gravity. Table 3.2 shows the speed of an object after it has fallen various distances. In Section 1.5 we illustrated the relationships between speed and time and between distance and time. This completes the picture.

We have the reverse situation when an object is thrown or projected straight up. It starts with kinetic energy, rises until all of that has been converted into potential energy, and then falls (Figure 2.20). The conservation of energy tells us that the kinetic energy that it begins with equals its potential energy at the highest point. This results in the same equation relating the initial speed \(v\) to the maximum height reached, \(d\):

\[
v^2 = 2gd
\]

which we can rewrite as:

\[
d = \frac{v^2}{2g}
\]

To compute how high something will go when thrown straight upward, insert its initial speed for \(v\) in this equation. We can also use Table 3.2 "backwards": a ball thrown upward at 12 mph will reach a height of 5 feet.

Now let's consider a similar problem. A roller coaster starts from rest at a height \(d\) above the ground. It rolls without friction or air resistance down a hill (see Figure 3.28). What is its speed when it reaches the bottom?

Again, the only forms of energy are gravitational potential energy and kinetic energy, since we assume there is no friction. The total energy, which is kinetic energy + potential energy, is constant. The kinetic energy of the roller coaster at the bottom of the hill must equal its potential energy at the top.

\[
KE \text{ (at bottom)} = PE \text{ (at top)}
\]

\[
\frac{1}{2}mv^2 = mgd
\]
As a roller coaster travels down a hill, its potential energy is converted into kinetic energy. If there is no friction, its kinetic energy at the bottom equals its potential energy at the top. Its speed at the bottom is the same as that of an object dropped from the same height.

\[ v = \sqrt{2gd} \]

It would have been impossible to solve this problem using only the tools from Chapter 2. To use Newton's second law, \( F = ma \), one needs to know the net force that acts at every instant. The net force pulling the roller coaster along its path varies as the slope of the hill changes, and so it is a very complicated problem. The principle of energy conservation allows us to easily solve a problem that we could not have done before. In the process we also come up with the following general result: The law of conservation of energy tells us that for an object affected by gravity but not friction, the speed that it has at a distance \( d \) below its starting point is given by the preceding equation regardless of the path it takes. The speed of a roller coaster that rolls down a hill is the same as that of an object that falls vertically the same height. The roller coaster does take more time to build up that speed (its acceleration is smaller), and the falling body therefore reaches the ground sooner.

The motion of a pendulum involves the continuous conversion of gravitational potential energy into kinetic energy and back again. Let's say that a child on a swing is pulled back (and up; see Figure 3.29). The child then has gravitational potential energy because...
of his or her height above the rest position of the swing. When released, the child swings downward, and the potential energy is converted into kinetic energy. At the lowest point in the arc, the child has only kinetic energy, which equals the original potential energy. The child then swings upward and converts the kinetic energy back into potential energy. This continues until the swing stops at a point nearly level with the starting point. This process is repeated over and over as the child swings. Air resistance takes away some of the kinetic energy. If the child is pushed each time, the work done puts energy into the system and counteracts the effect of the air resistance. Without air resistance or any other friction, the child would not have to be pushed each time and would continue swinging indefinitely.

The maximum height that a pendulum reaches (at the turning points) depends on its total energy. The more energy a pendulum has, the higher the turning points. In Section 3.2 we described a way to measure the speed of a bullet or a thrown object (Figure 3.6). The law of conservation of linear momentum is used to relate the speed of the bullet before the collision to the speed of the block (and bullet) afterwards. If the wood block is hanging from a string, the kinetic energy it gets from the impact causes it to swing up like a pendulum. The more energy it gets, the higher it will swing. We can determine the speed of the block after impact by measuring how high the block swings. The potential energy of the block (and bullet) at the high point of the swing equals the kinetic energy of the block (and bullet) right after impact. This results in the same equation relating the speed at the low point to the height reached.

\[ v = \sqrt{2gd} \]

In Section 2.6 we discussed the motion of an object hanging from a spring (Figure 2.24). The motion also consists of a continual conversion of potential energy into kinetic energy and back again.

- Figure 3.30 shows someone putting a ball at a miniature golf course. Since the ball rests in a small valley below ground level, its potential energy is negative relative to the level ground. When the ball is not moving, its total energy is negative because its kinetic energy is zero and its potential energy is negative. The golfer gives the ball kinetic energy by hitting it with a club. A weak putt gives it enough energy to roll back and forth but not "escape" from the hole (a). The ball's total energy is larger but still negative. In (b) the golfer gives the ball more energy by hitting it harder, but since its total energy is still negative, the ball again oscillates back and forth, although reaching a higher point on each side before turning around. In (c) the golfer hits the ball just hard enough for the ball to roll out of the valley and stop once it is out. The ball is given just enough kinetic energy to make its total energy equal to zero, and it "escapes" from the little valley. If the ball were hit even harder, it would escape and have excess kinetic energy—it would continue to roll on the level ground.

- **Figure 3.30** A golf ball at rest in the small valley has negative potential energy. Hitting the golf ball gives it kinetic energy, but it oscillates inside the valley if its total energy is negative, (a) and (b). If the golf ball is given enough kinetic energy to make its total energy zero, it rolls out of the valley and stops (c).
Energy Conservation, Consumption, and Crisis

We hear a lot about the need to conserve resources, particularly energy, in the news these days. In the light of our discussion in this chapter, one might wonder why people are concerned about conservation of energy; after all, energy must be conserved according to fundamental physical principles, mustn’t it? What’s all the fuss about something that happens naturally? The paradox here derives from the ways physicists and the general public use and interpret the phrase “energy conservation,” and it is an example of the kind of confusion that can arise when the same words are used with different meanings by different groups.

As we have seen in Section 3.5, conservation of energy as a physical law refers to the fact that energy can neither be created nor destroyed in any interaction, but merely changed in form. Put another way, in any system the total amount of energy remains constant. However, conservation of energy as an economic, environmental, or social principle involves reducing our reliance on certain types of energy sources like coal, oil, and natural gas, and becoming more efficient in their use in cases where switching to other sources is deemed unacceptable and/or unfeasible. For the general public, then, conservation of energy means husbanding precious “nonrenewable” natural resources and, overall, using less energy to accomplish the myriad tasks we carry out daily in our society.

There is no doubt, however, that one important aspect of the public’s interpretation of the phrase “energy conservation” relates to physicists and engineers. Specifically, if we are to develop new processes and devices that use energy more efficiently (i.e., accomplish more useful work with less output of waste heat), such improvements must be achieved within the bounds of the known and accepted laws of physics, including the law of conservation of energy. No process, for example, can create energy where there was none beforehand (although no device can be made 100% efficient in the conversion of one form of energy into another. For more on the topic of energy conversion, see Section 5.7.) Technological elements of energy conservation, as commonly understood by the public, require knowledge and application of the fundamental laws of physics.

But what has driven the public to be the least bit concerned about energy conservation, much less its connection to physics? Among the elements that have prompted society’s attempts to use less energy is the way energy consumption, especially in the United States, has increased over the last 150 years. Figure 3.31 displays the annual amount of energy consumed in the United States since 1850 from sources like wood, coal, oil, and natural gas, as well as from hydroelectric, geothermal, and nuclear power generating plants. The principal factor responsible for this growth in energy consumption has been the increase in the population of our country, at least over the last 140 years. But the point of interest for us in this graph is the manner in which the increase in energy consumption has occurred and not so much its cause.

Note that the graph is not a straight line. From one year to the next, the amount of energy consumed does not increase by a fixed amount as would be the case if the relationship between energy use and time were a linear one—that is, if there were a direct proportionality between these two quantities. Instead, energy consumption grows increasingly rapidly with each passing year. If one analyzes this behavior carefully, it may be demonstrated that the change in energy consumption during any given interval of time depends directly on the amount of energy being used at the start of the time interval. Symbolically, using the notation of Chapter 1, we may write

$$\Delta E/\Delta t \propto E$$

where $E$ is the energy consumption, $\Delta$ (a Greek capital delta) means “change in,” and $\propto$ is the mathematical symbol for “proportional to.” A quantity that increases (or decreases) in this manner is said to exhibit exponential growth (or decay). Evidently, energy consumption in the United States has grown exponentially since 1850, and one may ask whether this trend should cause us any concern if allowed to continue. Many people would argue that the answer to this question is a resounding “Yes!” In the judgement of these investigators, the origins of the energy crisis are to be found in the exponential growth of energy consumption. To understand their reasoning, it is necessary to consider some additional properties of exponential functions.

When faced with a quantity, call it $X$, which is increasing in an exponential fashion, one of the questions that might be asked is, How long does it take for the amount of $X$ to double in size? By studying the mathematical character of expo-
ponential functions, it is possible to show that the so-called doubling time is related to the percent increase of the quantity in a given interval of time (a year, a week, a day, a second, etc.); specifically, if we let \( t_D \) represent the doubling time, then

\[
    t_D \approx \frac{70}{PC}
\]

where \( PC \) is the percent change over some unit of time. The units of time for \( t_D \) will be those used to express \( PC \). (The "\( \approx \)" in the formula means "approximately equal to." More precise computation reveals the numerator on the right-hand side to be 69.3, but rounding this number to 70 provides sufficient accuracy for many applications.) So, for example, if a certain quantity grew steadily at a rate of 10% per year, it would double in size in 70/10 or 7 years. Conversely, if one knew that something had increased by a factor of 2 in 25 minutes, then it would follow, assuming it varied exponentially, that it was growing at a rate of nearly 3% per minute.

Returning now to the issue of energy consumption in the United States, the data shown in Figure 3.31 yield a growth rate of a little over 4% a year, at least for many years preceding 1975. At this rate, we find that the doubling time for energy use in this country would be about 16 years. This means that if such growth were sustained, every 16 years the amount of energy consumed in the United States would double in size, requiring a similar doubling of the production capabilities of all the energy industries in the nation. Similarly, at this rate, in only 80 years (five doubling times), the U.S. energy consumption (and production) would increase by a factor of \( 2 \times 2 \times 2 \times 2 = 2^5 \) or 32. As may be seen, exponential quantities rise (or fall) extremely quickly, even for what might be considered modest growth (or decay) rates. If we push this example just a bit farther, we can begin to see why scientists, engineers, economists, government officials, and others have called for profound changes in the way our society uses energy to avoid a severe "energy crisis": At 4% annual growth, in just 320 years the amount of energy required by our citizenry would be over 1 million times what it is today! This is far beyond even the most optimistic estimates of our capacity to find and/or perfect new energy sources.

Clearly, for exponentially increasing quantities, as time goes on, their values quickly become enormous, eventually approaching infinity (see Table 3.3). For this reason, it is impossible for any real quantity to continue to grow exponentially for a long period of time, much less forever. The ability of any system to sustain such growth is rapidly outstripped, and the growth is halted. Since the 1970s, the public's appreciation of the severity of the energy problem has deepened considerably, as has its commitment to adopting measures to reduce the growth in energy consumption. In part because of energy conservation measures implemented during the past 20 years, particularly the production of more fuel-efficient cars, total energy consumption in the nation has slowed considerably.

### Table 3.3  Exponential Growth and Doubling Times

<table>
<thead>
<tr>
<th>Doubling Times</th>
<th>Growth Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 2^1 = 2 )</td>
</tr>
<tr>
<td>2</td>
<td>( 2^2 = 4 )</td>
</tr>
<tr>
<td>3</td>
<td>( 2^3 = 8 )</td>
</tr>
<tr>
<td>4</td>
<td>( 2^4 = 16 )</td>
</tr>
<tr>
<td>5</td>
<td>( 2^5 = 32 )</td>
</tr>
<tr>
<td>10</td>
<td>( 2^{10} = 1,024 )</td>
</tr>
<tr>
<td>15</td>
<td>( 2^{15} = 32,768 )</td>
</tr>
<tr>
<td>20</td>
<td>( 2^{20} = 1,048,576 )</td>
</tr>
<tr>
<td>25</td>
<td>( 2^{25} = 3.3 \times 10^8 )</td>
</tr>
<tr>
<td>50</td>
<td>( 2^{50} = 1.1 \times 10^{15} )</td>
</tr>
</tbody>
</table>

Lest you conclude that only energy can suffer the type of "crisis" that can result from exponential behavior, it should be mentioned that many familiar quantities vary exponentially: Savings accounts that have fixed interest rates and are compounded continuously grow exponentially (a fact that led to the passage of banking laws that limit the amount of time an account may be left untouched); biological populations, including human ones, have been shown to rise exponentially for certain periods during their development (until limitations on space and food force an end to such steady growth); the number of radioactive nuclei remaining in a sample of material decays with time according to an exponential curve (for which it becomes of interest to know the time during which the number of nuclei is reduced by half, the half-life of the sample; see Section 11.3 for more details on this application); and the amount of Internet traffic on the World Wide Web has continued to grow exponentially with a doubling time of about one year since 1991. Even information can grow in an exponential fashion as molecular biologists who use large genetic and protein structure databases are discovering. For example, the size of the Protein Data Bank which maintains three-dimensional models of important biological macromolecules is doubling every 18 months, forcing researchers and data managers to develop faster and more efficient computer programs to store, validate, and retrieve the information. The appearance of exponentials in the affairs of humankind is widespread and, yet, as someone has commented: "The greatest shortcoming of the human race is our inability to understand the exponential function." Hopefully, this short discussion has given you a greater appreciation for the importance and the potential dangers of exponential functions. Perhaps it may also make you somewhat more skeptical about the alleged virtues of unfettered growth, whether in physics, finances, families, or factories.
This is the principle behind rocking a car when it is stuck. If a tire is in a hole, it is best
to make the car oscillate back and forth. By giving it some energy during each cycle, by
pushing or by using the engine, one can often give the car enough energy to leave the hole.

There are many analogous systems in physics in which an object is bound unless its
total energy is equal to or greater than some value. A satellite in orbit about the Earth is an
important example. The satellite’s motion from one side of the Earth to the other and back
is similar to the motion of the golf ball in the valley. If it is given enough energy, the satel-
lite will escape from the Earth and move away, much like the golf ball. The minimum speed
that will give a satellite enough energy to leave the Earth is called the escape velocity. Its
value is approximately 11,200 m/s or 25,000 mph.

When water boils, the individual water molecules are given sufficient energy to break free
from the liquid (Chapter 5). Sparks and lightning occur only after electrons are given enough
energy to break free from their atoms (Chapter 7). The transition of a system from a bound
state to a free state is quite common in physics and is not limited to mechanical systems.

These examples illustrate the qualitative and quantitative usefulness of the law of con-
servation of energy. It allows us to treat some systems that we couldn’t before, and it pro-
vides another way of looking at familiar systems.

**Learning Check**

1. To be able to do work, a system must have

2. An object has kinetic energy when it is

3. As a skier gains speed while gliding down a slope,
   ____________________ energy is being converted into ____________________
   energy.

4. (True or False.) Work done inside an isolated system can increase the total energy in the system.

5. (Choose the incorrect statement.) The total kinetic energy plus potential energy of a body
   a) can be negative.
   b) always remains constant if the body is freely falling.
   c) always remains constant if friction is acting.
   d) can remain constant even if the body’s speed is decreasing.

**Answers:** 2. kinetic 3. potential, kinetic

### 3.6 Collisions: An Energy Point of View

Earlier in this chapter we pointed out that the main “tool” for studying all collisions is the
law of conservation of linear momentum (Section 3.2). In this section we look at collisions
from an energy standpoint. In some collisions, the only form of energy involved, before
and after, is kinetic energy. In other collisions, forms of energy like potential energy and
internal energy play a role. Collisions can be classified as follows.

**An Elastic Collision** is one in which the total kinetic energy of the colliding
bodies after the collision equals the total kinetic energy before the collision.

**An Inelastic Collision** is one in which the total kinetic energy of the colliding
bodies after the collision is not equal to the total kinetic energy before. The total
kinetic energy after can be greater than, or less than, the total kinetic energy before.

In an elastic collision, kinetic energy is conserved. The total energy is always conserved
in both types of collisions, but in elastic collisions no energy conversions take place that
make the total kinetic energy after different from the total kinetic energy before.

◆ Figure 3.32 illustrates examples of these two types of collisions. Two equal-mass
carts traveling with the same speed but in opposite directions collide. In both collisions,
the total linear momentum before the collision equals the total after. (This total is equal to
zero. Why?) In (a) the carts bounce apart because of a spring attached to one of them. Af-
ter the collision, each cart has the same speed it had before, but it is going in the opposite
direction. Consequently, the total kinetic energy of the two carts is the same after the col-
losion as it was before. This is an elastic collision.

Figure 3.32b is an example of an inelastic collision. This time the two carts stick to-
gether (because of putty on one of them) and stop. The total kinetic energy after the col-